Question Booklet Series: **A** *Question Booklet Serial No.* 104526

M. Phil./Ph. D. Entrance Test – 2016

Subject: Statistics

Paper – I

Important: *Please consult your Admit Card/Roll No. slip before filling your Roll Number on the Test Booklet and Answer Sheet.*

Roll No. ***In Figure*** ***In Words***

O.M.R. Answer Sheet Serial No.

Signature of Candidate: ______________ *Signature of Invigilator:* ______________

Time: 60 Minutes Number of Questions: 50 Maximum Marks: 50

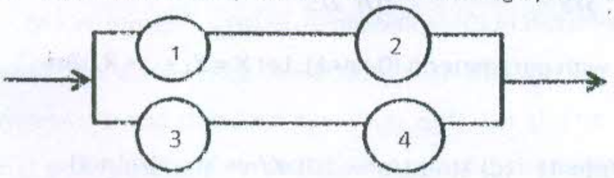
DO NOT OPEN THE SEAL ON THE BOOKLET UNTIL ASKED TO DO SO.

INSTRUCTIONS:

1. Write your Roll No. on the Questions Booklet and also on the OMR Answer Sheet in the space provided and nowhere else.
2. Enter the Question Booklet Serial No. on the OMR Answer Sheet. Darken the corresponding bubbles with **Black Ball Point/Black Gel Pen**.
3. Do not make any identification mark on the Answer Sheet or Question Booklet.
4. Please check that this Question Booklet contains 50 Questions. In case of any discrepancy, inform the Assistant Superintendent within 10 minutes of the start of Test.
5. Each question has four alternative answer (A,B,C,D) of which only one is correct. For each question, darken only one bubble (A or B or C or D), whichever you think is the correct answer, on the Answer Sheet with **Black Ball Point/Black Gel Pen.** **<u>There shall be no negative marking for wrong answers.</u>**
6. If you do not want to answer a question, leave all the bubbles corresponding to that question blank in the Answer Booklet. No marks will be deducted in such cases.
7. Darken the bubbles in the OMR Answer Sheet according to the Serial No. of the question given in the Question Booklet.
8. If you want to change an already marked answer, erase the shade in the darkened bubble completely.
9. For rough work only the blank sheet at the end of the Question Booklet be used.
10. The Answer Sheet is designed for computer evaluation. Therefore, if you do not follow the instructions given on the Answer Sheet, it may make evaluation by the computer difficult. **Any resultant loss to the candidate on the above account, i.e. not following the instructions completely, shall be of the candidate only.**
11. After the test, hand over the Question Booklet and the Answer Sheet to the Assistant Superintendent on duty.
12. In no case the Answer Sheet, the Question Booklet, or its part or any material copied/noted from this Booklet is to be taken out of the examination hall. Any candidate found doing so would be expelled from the examination.
13. A candidate who creates disturbance of any kind or changes his/her seat or is found in possession of any paper possibly of any assistant or found giving or receiving assistant or found using any other unfair means during the examination will be expelled from the examination by the Centre Superintendent/Observer whose decision shall be final.
14. **Communication equipment such as mobile phones, pager, wireless set, scanner, camera or any electronic/digital gadget etc., is not permitted inside the examination hall. Use of calculators is not allowed.**
15. The candidates will not be allowed to leave the Examination Hall/Room before the expiry of the allotted time.

1. Let $X \geq o$ almost surely. Suppose $E(X^{-1})$ and $E(X^{\alpha})$ exist, where $\alpha > 0$. Then $E(X^{\alpha})\, E(X^{-1})$ is:
(A) $\leq E(X^{\alpha-1})$ (B) $\geq E(X^{\alpha-1})$ (C). $= E(X^{\alpha-1})$ (D) $= E(X^{\alpha})/E(X)$

2. Each of the components in the following system functions independently with probability .6.



The probability that the system functions adequately is:
(A) .72 (B) .1296 (C) .5904 (D) .28

3. Let X and Y be random variables. The inequality $(E(X+Y)^2)^{1/2} \leq (E(X^2))^{1/2} + (E(Y^2))^{1/2}$
(A) Does not hold (B) Holds for any X and Y (C) Holds only if X and Y are non-negative (D) Holds only if X and Y are degenerate.

4. The value of k for which the function $f(x, y) = kx(x-y)$, $0<x<1$, $-x<y<x$, is a joint density is:
(A) 2 (B) 1 (C) 24 (D) 4

5. Three items drawn from a lot containing 5 items were found defective. The probability that all the items in the lot were defective is:
(A) 4/5 (B) 1/2 (C) 2/5 (D) 2/3

6. The joint probability density function (pdf) of (X,Y) is $f(x,y) = 2$, $0<x$, $0<y$, $x+y<1$. The value of $P(X+Y>1/2)$ is:
(A) 2/3 (B) 1/2 (C) 4/9 (D) 5/9

7. A simple random sample of size 6 was drawn with replacement from a lot containing 5 defective and 10 non-defective items. Let random variable X denote the number of non-defective items in the sample. The value of E(X) is:
(A) 2 (B) 5 (C) 6 (D) 4

8. Let $X_1, \ldots, X_n$ be a random sample from uniform distribution over the interval (0, 1).Let random variable N_i denote the number of sample observations in the interval (a_i, b_i), where $a_i = (i-1)/k$, $b_i = a_i+1/k$, $i = 1, \ldots, n$, $k>0$. Covariance between N_3 and N_8 is:
(A) n/k^2 (B) $-1/k^2$ (C) $-n/k^2$ (D) nk^2

A single sampling plan with c=14 is used by taking a sample of size 225 items from a lot consisting of 2200 items. The probability of accepting this lot is .83 when the lot quality is .05. Answer questions 9 and 10 using this information.

9. The average total inspection (ATI), rounded off to nearest whole number, is:
(A) 110 (B) 1826 (C) 187 (D) 561

10. The value of average out going quality (AOQ) is:
(A) .0415 (B) .558 (C) .110 (D) .05

11. A simple random sample of 4 items was drawn without replacement from a lot containing 4 defective and 6 non-defective items. Let random variable X denote the number of non-defective items in the sample. The value of E(X) is:
(A) 3.6 (B) 2.4 (C) 1.6 (D) 2

12. Let random variable X follows Uniform distribution over the interval (0, 4). The value of $P(X>2 \mid 1<X<3.5)$ is:

(A) 3/8 (B) 1/2 (C) 3/5 (D) 2/5

13. Let $X_1, \ldots, X_n$ be iid Bernoulli variates with parameter p (0< p<1). Let $X = X_1 + \ldots + X_n$. An UMVUE of p + p(1-p) is:

(A) $X/n+X(X-n)/n^2$ (B) $X(n-X)/n(n-1)$ (C) $X(n-X)/n^2$ (D) $X/n+X(n-X)/n(n-1)$

14. The basic purpose of bootstrap method of estimation is to:

(A) Decrease Bias (B) Decrease Variance (C) Decrease MSE (D) Increase ARE

15. Let X_1 and X_3 be iid standard exponential random variables. The value of $P(X_1 > X_2)$ is:

(A) 3/4 (B) 1/4 (C) 2/3 (D) 1/2

16. In stratified simple random sampling the Variance of stratified sample mean is minimum for fixed total sample size n if $n_i \alpha N_iS_i$. The value of constant of proportionality is:

(A) nS_i/N (B) $N_iS_i/\sum N_i$ (C) $N_iS_i/\sum S_i$ (D) $N_iS_i/\sum N_iS_i$

17. Let $T_{(r)}$ be the rth order statistic associated with a random sample of size n from an exponential distribution with mean θ. The value of $E(T_{(r)}/\theta)$ is:

(A) rθ (B) θ/n-r+1 (C) $\sum_{i=1}^{r} 1/(n-i+1)$ (D) $\theta \sum_{i=1}^{r} 1/(n-i+1)$

18. In cluster sampling, we take random sample:

(A) of clusters alone (B) from each cluster (C) of clusters and then take samples from selected clusters (D) from entire population

19. Let r be the number of components which fail before fixed time T^o out of n components under Type-1 censoring. Let T be the total life time observed and that the life times of components are iid exponential random variables with mean θ. The minimal sufficient statistic for θ is:

(A) (r, n) (B) (n, T) (C) (n, r, T) (D) (r, T)

20. Let Y and X, respectively, be the study and auxiliary variables. We prefer ratio estimator if Y is:

(A) Independent of X (B) non-linearly related to X (C) proportional to X (D) proportional to a linear function of X

21. Let $X_1, \ldots, X_n$ be a random sample from an exponential distribution with location parameter μ and scale parameter θ. Let T= min ($X_1, \ldots, X_n$) and $S = \sum_{i=1}^{n} (X_i - T)/(n-1)$. The distribution of 2n (T-μ)/S is:

(A) Chi-square (B) F (C) Gamma (D) Beta

22. Let X be a random variable such that $E(e^{aX})$ exists for a>0. Then $P(X \geq \tau)$ is:

(A) $\leq E(e^{aX})/ e^{a\tau}$ (B) $\geq E(e^{aX})/e^{\tau}$ (C) $\geq E(e^{aX})/ e^{a\tau}$ (D) $\leq e^{a\tau} E(e^{aX})$

23. Let $\{X_i\}$ be a sequence of independent random variables such that $E(X_i)=\mu_i$ and $var(X_i)=\sigma_i$, I= 1, ..., n.Define $Y_n = (X_1+...+X_n)/n$ and $\delta_n = (\mu_1+...+\mu_n)/n$. If $\sum_{i=1}^{n} \sigma_i / n^2 \to 0$ as $n \to \infty$ then Y_n converges to δ_n :

(A) in mean (B) in probability (C) in distribution (D) almost surely.

24. Let $\{T_n\}$ be a sequence of statistics such that $\sqrt{n}(T_n - \theta)$ follows normal distribution with mean zero and standard deviation $\sigma(\theta)$. Let g(x) be a function of single variable admitting first derivative $g'(x)$. The asymptotic distribution of $\sqrt{n}(g(T_n) - g(\theta))/\sigma(\theta)$ is normal with variance:

(A) 1 (B) $g'(\theta)$ (C) $(g'(\theta))^2$ (D) $g'(\theta)/(\sigma(\theta))^2$

25. Let A be the set of elements common to infinite number of events of the sequence of events $\{A_n\}$. Then $\sum P(A_n) < \infty$ implies that the value of P(A) is:

(A) 1 (B) 0 (C) in the interval (0, 1) (D) greater than 0

26. Let X be any random variable with $E(X) < \infty$. Then E(X) is

(A) $\le \log_e(E(e^X))$ (B) $\ge \log_e(E(e^X))$ (C) $e^{E(X)} \ge E(e^X)$ (D) $< \log_e(e^{E(X)})$

27. The probability mass function (pmf) of a discrete random variable X is $p(x) = 2^{-x}$, x = 1, 2, ... The value of E(X) is:

(A) 1/2 (B) 2 (C) not defined (D) 4

28. The joint probability density function of (X,Y) is f(x,y) = ¼ , $|x| < 1$, $|y| < 1$. The value of $P[X^2 + Y^2 \le 9/16]$ is:

(A) 3/4 (B) 3/16 (C) 9/64 (D) 9/16

29. The joint pdf of (X,Y) is $f(x,y) = \{e^{-(x+y)}x^3y^4\}/144$, x>0, y>0. Define the random variables U= X/(X+Y) and V= X+Y. The Jacobian of transformation to find the joint pdf of (U,V) is:

(A) u (B) 1 (C) v (D) uv

30. Let $N_1(t)$ and $N_2(t)$ be two independent Poisson processes with rates τ_1 and τ_2 , respectively. The conditional distribution of $N_2(t)$ given $N_1(t) + N_2(t) = n$ is:

(A) Binomial (B) Geometric (C) Hypergeometric (D) Negative binomial

31. Let X and Y be two random variables with cdf $F_X(x) = F(x-\mu_X)$ and $G_Y(y) = F(y-\mu_Y)$, where F is any continuous distribution independent of parameters. If we write $G_Y(x) = F_X(x-\theta)$ then θ is equal to:

(A) $\mu_X - \mu_Y$ (B) $\mu_Y - \mu_X$ (C) μ_X / μ_Y (D) μ_Y / μ_X

32. Let X and Y be two random variables with cdf $F_X(x) = F(x/\sigma_X)$ and $G_Y(y) = F(y/\sigma_Y)$, where F is any continuous distribution independent of parameters. If we write $G_Y(x) = F_X(x\theta)$ then θ is equal to:

(A) $\sigma_X . \sigma_Y$ (B) $\sigma_Y - \sigma_X$ (C) σ_X / σ_Y (D) σ_Y / σ_X

33. Let $S \sim W(k, \Sigma)$. For fixed vector L the distribution of $(L'\Sigma^{-1}L)/(L'S^{-1}L)$ is:

(A) $W(k-p, \Sigma)$ (B) $\chi^2(k-p)$ (C) $\chi^2(k-p+1)$ (D) $W(k-p-1, \Sigma)$

34. Let $S \sim W(k, \Sigma)$ and consider the partition of S as $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$, where S, S_{11}, S_{12} and S_{22} are (pxp), (rxr), (rxs) and (sxs) matrices with r+s=p. Consider similar partition of Σ as Σ_{11}, Σ_{12} etc. The distribution of $S_{22} - S_{21}S_{11}^{-1}S_{21}$ is:

(A) Chi-square with s df (B) Gamma with shape p (C) Beta (r,s) (D) Wishart with s df

35. In constructing 1/6 fraction of $2^3 x 3^3$ design suppose we take ½ fraction of a 2^3 experiment by confounding ABC and a 1/3 fraction from a 3^3 experiment by confounding DE^2F and D^2EF^2. The total number of confounded contrasts will be:

(A) 3 (B) 4 (C) 5 (D) 7

36. In a BIBD the number of experimental plots required is:

(A) vk (B) bk (C) vr (D) kr

37. All contrasts of treatments effects are estimable in a design if and only if the design is :

(A) balanced (B) connected (C) orthogonal (D) complete.

38. The transition probability matrix (TPM) of a Markov Chain $\{X_n\}$, n = 1,2, ... with three states 1,2,3 is

$$P = \begin{bmatrix} .1 & .5 & .4 \\ .6 & .2 & .2 \\ .3 & .4 & .3 \end{bmatrix}$$

with initial probabilities as (.7 .2 .1). The value of $P(X_2 = 3, X_1 = 3, X_0 = 2)$ is:

(A) .2 (B) .04 (C) .012 (D) .09

39. Let μ_{ii} and d_i, respectively, be the mean recurrence time and period of state i of a Markov Chain. The state I is said to be ergodic if:

(A) $\mu_{ii} < \infty$ (B) $d_i > 1$ (C) $\mu_{ii} = \infty$, $d_i > 1$ (D) $\mu_{ii} < \infty$, $d_i=1$

40. The value of k for which the function f(x) = k is a uniform density in the region bounded by curves y = x and $y = x^2$ for $0 < x < 1$ is

(A) 2 (B) 6 (C) 4 (D) 1/6

41. Let P = (p_{ij}) be the TPM of a Markov Chain. This chain is said to be irreducible if:

(A) $p_{ij} > 0$ for all $\forall i, j$ (B) $p_{ij}^{(n)} > 0\ \forall i, j$ and some n (C) $\sum_i p_{ij}^{(n)} = 1$ for some n

(D) $\sum_j p_{ij}^{(n)} < 1$ for $\forall i$ and some n.

42. The probability distribution of random variable N, denoting the number of items in the shipment, is:

n (value of N) :	10	15	20	25
P(N=n) :	.30	.20	.30	.20

The probability is .10 that any shipped item is defective. Let random variable X denote the number of defective items shipped each day. The expected value of X is

(A) 17 (B) 1.725 (C) .17 (D) 1.7

43. Let $U_1, \ldots, U_n$ be iid standard uniform random variables. Define $Y_i = U_i / \sum_{i=1}^{n} U_i$, I = 1, ..., n.

The random variables $Y_1, \ldots, Y_n$ are:

(A) Dependent (B) Linearly dependent (C) Independent (D) Undefined

44. Two stage sampling is a compromise between two sampling schemes, namely:

(A) Stratified and simple random (B) stratified and cluster (C) stratified and systematic (D) stratified and pps.

45. Let $Y_1, \ldots, Y_n$ be independent random variables such that $E(Y_i) = \mu_i$, $Var(Y_i) = \sigma^2$, i = 1, ..., n. Let tr(A) be the trace of matrix A, $Y' = (Y_1, \ldots, Y_n)$ and $\mu' = (\mu_1, \ldots, \mu_k)$. Then $E(Y'AY)$ is:

(A) $\sigma^2 tr(A) + \mu' A \mu$ (B) $\sigma^2 tr(A)$ (C) $\mu' A \mu$ (D) $\sigma^2 \mu' tr(A) \mu$

46. Let $X_1, \ldots, X_{20}$ be a random sample from a continuous distribution with median M and variance 4. Let random variable Y denote the number of sample observations greater than M. The variance of random variable Y is:

(A) 10 (B) 5 (C) 20 (D) 40

47. Let f(x) = x^2 and g(x) = e^{-x}. The value of $\int_0^{\infty} f(x)dg(x)$ is:

(A) -2 (B) 2 (C) 1 (D) 0dx

48. Let $F_n(x)$ be the empirical distribution function associated with a random sample $X_1, \ldots, X_n$ from a continuous distribution with cumulative distribution function (CDF) F(x). The variance of $F_n(x)$ is:

(A) F(x)(1-F(x))/2 (B) F(x)(1-F(x)) (C) nF(x)(1-F(x)) (D) nF(x)/4

49. Let $F^{-1}(u) = \theta_1 + \theta_2^{-1}(u^{\theta_3} - (1-u)^{\theta_4})$, 0<u<1, be the inverse of a CDF, where θs are all positive parameters. The distribution is symmetric if:

(A) $\theta_3 > \theta_4$ (B) $\theta_3 = \theta_4$ (C) $\theta_1 = \theta_2$ (D) $\theta_3 < \theta_4$

50. Let f(x) be the pdf associated with the cdf F(x) of a continuous non-negative random variable X. This distribution is IFR if $\log_e f(x)$ is:

(A) Convex (B) Linear (C) Concave (D) Non-ninear.